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REMARKS ON THE PASSAGE THROUGH RESONANCE OF A VIBRATING SYSTEM WITH TWO DEGREES OF FREEDOM, EXCITED BY A NON-IDEAL ENERGY SOURCE.

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1. PRELIMINARY COMMENTS

The interaction between a dynamic system and a non-ideal source of disturbance was discovered experimentally at the beginning of the century by Sommerfeld [1] and was later called "Sommerfeld's effect". The first attempt at an analytical research of this phenomenon was made by Rocard [2] and was published in 1949. In 1953, Blekhman, while investigating the self-synchronization of unbalanced rotating masses, came to the conclusion that the vibrating system passes from a resonant to a non-resonant regime in a jump-like manner [3] (jump phenomenon). In this research, the torque generated by the motor is assumed to be proportional to the angular velocity of rotation of the rotor. In 1958, Kononenko [4], while studying the behavior of a dynamic system with single degree of freedom (s.d.o.f.) and a non-ideal disturbance source, used the static characteristics of the energy source in the form of a non-linear function of the torque generated by the motor and the angular velocity of the rotor. For the same dynamic system, in reference [5], the passage through resonance

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was investigated, taking into account its interaction with the non-ideal energy source. The principles of the theory of vibration of systems with a limited power supply are introduced in reference [6] in which it is demonstrated that, with a limited driving power, the angular velocity of the vibrator cannot be a random one; it is determined by the interaction between the system and the vibrator. The Sommerfeld effect (and its properties) in the non-ideal problem is described in the book by Kononenko [7] entirely devoted to this subject. That experiment, carried out in 1904, detected all the effects of interactions between a non-ideal motor and its elastic foundation. Several other experiments are mentioned in that same book and in the one by Nayfeh and Mook [8]. A complete and comprehensive review of different theories of non-ideal vibrating systems is given in references [9, 10]. An interesting case occurs when the dynamic system is electromechanical; i.e., when the torque generated by the motor is determined by the dynamic characteristics of the motion. In references [11, 12] we analyze the behavior of a mathematical pendulum vibrating in a horizontal plane suspension point and a non-ideal energy source (DC motor). The relation between the torque generated by the motor and the angular velocity of the rotor is determined by the dynamic characteristics of the motor. This problem was proposed before in reference [13]. In reference [14] the authors analyzed possible slip–stick motions of the non-ideal, self-excitation problem. In references [15–17] the authors analyzed experimentally a shaft on a console with a DC motor with an unbalanced rotor placed at the free end of the shaft. The dynamic system includes square and cubic non-linearity. The control of the motor and, accordingly, the change of the angular velocity are achieved by change of voltage. The possibility of a controlled passage through resonance in dynamic systems with an ideal energy source is investigated in the book by Chernousko *et al.* [18] and the papers by Nagaya *et al.* [19], and Wauer [20], in which the control of the vibrations is realized by switching the elastic components of the suspension structure. In references [20, 21] and Dimentberg *et al.* in references [22, 23] examine a controllable passage in a system with an s.d.o.f. and a non-ideal source of energy. The control of the system is realized by switching the elastic components in the suspension of the main mass. The dependence of the torque generated by the motor on the angular velocity of the rotor is determined by the static characteristics of the DC motor. In reference [24] the authors discussed the optimum design of an operating curve for a rotating shaft system with a limited power supply using a gradient-based optimization method. A problem with 2d.o.f. subject to a non-ideal motor was treated in references [25, 26]. In these works, the authors studied the non-linear vibrations of a multiple machines portal frame foundations. Two unbalanced rotating machines are considered, none of them resonant with the lower natural frequencies of the supporting structure. Their combined frequencies are set in such a way as to excite, due to non-linear behavior of the frame, either the first antisymmetrical mode (sway) or the first symmetrical mode. The physical and geometrical characteristics of the frame are chosen to tune the natural frequencies of these two modes into a 1 : 2 internal resonance. The problem is reduced to a 2d.o.f. model. In reference [27] a dynamic system with 2d.o.f., subjected to a dynamic disturbance with a limited power supply, is investigated. The dependence of the moment generated by the motor on the angular velocity of the rotor is partly a linear function. The case in which the parameters of the system are chosen in such a way that the amplitude frequency characteristic has two resonant peaks and the stationary motion is in the antiresonance region is analyzed. The parameter space of the moment generated by the motor in which the system performs subharmonic and quasiperiodic vibrations is determined. Defining the parameters at which determinate chaos occurs is a different research problem.

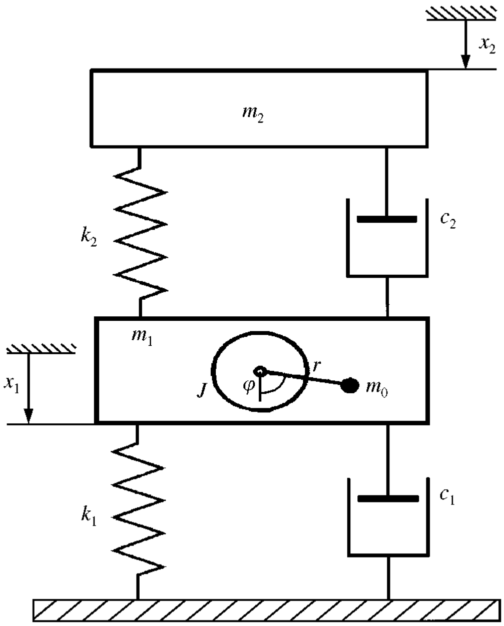

Figure 1. The problem.

2. DESCRIPTION OF THE SYSTEM

The model of the vibrating system and the source of disturbance with a limited power supply is illustrated in Figure 1. The vibrating system consists of a mass m_1, a linear elastic spring with a coefficient of elasticity k_2 and a coefficient of damping c_1. On the object with mass m_1, a non-ideal power supply source (DC motor) is placed, with a driving rotor of a moment of inertia J and an eccentrically situated mass m_0 at a distance r from the axis of rotation. By means of a linear spring with a coefficient of elasticity k_2 and a damper with a coefficient of damping c_2 an object of mass m_2 has been attached to mass m_1. The differential equations of motion of the system are [27]

$$m_1\ddot{x}_1 = -k_1x_1 - c_1\dot{x}_1 + k_2(x_2 - x_1) + c_2(\dot{x}_2 - \dot{x}_1) + m_0r\omega^2\cos\varphi + m_0r\dot{\omega}\sin\varphi,$$
$$m_2\ddot{x}_2 = -k_2(x_2 - x_1) - c_2(\dot{x}_2 - \dot{x}_1), \quad (1)$$
$$J\dot{\omega} = L - H(\omega) + m_0\ddot{x}_1\sin\varphi, \quad \dot{L} = -aL - b\omega + k_UU(\omega),$$

where L is the torque generated by the motor and $H(\omega)$ is the resisting torque, which will be ignored from now on, a and b are constant values depending on the type and power of the motor, and $U(\omega)$ is the voltage of the motor. The last equation of the system shows the dynamic characteristics of the DC motor. The use of the dynamic characteristics is connected with the non-stationary nature of the processes going on in the vibrating system and the limited power supply source. Equations (1) include only non-linear members resulting from the interaction between the vibrating system and the energy source. The objective of this study is to synthesize optimal control of the passage through the first resonant peak. The presence of non-linear elastic components and non-linear dampers is not of great significance, and will therefore be ignored. The effect of these non-linearities will be the topic of another research. In order to simplify the solutions of the equations and the analysis of the results received we use dimensionless variables, introducing the parameters

$$\chi_1 = x_1 \frac{m_1}{m_0 r}, \quad \chi_2 = x_2 \frac{m_1}{m_0 r}, \quad \tau = t\sqrt{\frac{k_1}{m_1}}, \quad \bar{\omega} = \omega\sqrt{\frac{m_1}{k_1}}, \quad \mu = \frac{m_2}{m_1}$$

$$\eta_1 = c_1\sqrt{\frac{1}{k_1 m_1}}, \quad \eta_2 = c_2\sqrt{\frac{1}{k_1 m_1}}, \quad \theta^2 = \frac{m_1 k_2}{m_2 k_1}, \quad \rho = \frac{(m_0 r)^2}{J m_1}, \tag{2}$$

$$\lambda = L\frac{m_1}{J k_1}, \quad \alpha = a\sqrt{\frac{m_1}{k_1}}, \quad \beta = b\frac{m_1}{J k_1}, \quad u = k_U U \frac{m_1}{J k_1}\sqrt{\frac{m_1}{k_1}}.$$

Substituting equations (2) into equations (1) we obtain a system of differential equations which describe the motion of a linear dynamic system with a non-ideal energy source in dimensionless form

$$\ddot{\chi}_1 = -\chi_1 - \eta_1\dot{\chi}_1 + \mu(\chi_2 - \chi_1) + \mu\eta_2(\dot{\chi}_2 - \dot{\chi}_1) + \omega^2\cos\varphi + \dot{\omega}\sin\varphi,$$

$$\ddot{\chi}_2 = -\theta^2(\chi_2 - \chi_1) - \eta_2(\dot{\chi}_2 - \dot{\chi}_1), \tag{3}$$

$$\dot{\bar{\omega}} = \lambda + \rho\ddot{\chi}_1\sin\varphi, \quad \dot{\lambda} = -\alpha\lambda - \beta\bar{\omega} + u,$$

where the point above the variable indicates differentiating in the dimensionless time τ.

The system (3) is solved numerically by the method of Runge–Kutta with eighth order of precision and with an automatic choice of steplength [28]. At the initial time all state variables are zero and the initial value of the control function is chosen so that in the final moment the system performs a stationary motion at an average angular velocity of the rotor $\bar{\omega}_0 = 0.904698$.

3. FORMULATION OF THE PROBLEM

The problem of optimal control of the process of passage through resonance of a dynamic system with a non-ideal power supply is defined as follows. The functions χ_1, χ_2, $\bar{\omega}$, λ and control $u(\bar{\omega})$ minimizing the functional $I = T_{trans}$, satisfying the differential equations (3), and the bound $|u(\bar{\omega})| \leqslant 1.0$ should be determined in such a way that the dynamic system could pass from its initial position into a final position corresponding to the stationary motion in the antiresonance region. The objective function I is the time of the transitional process. Thus formulated, the problem of control of the passage is a problem of terminal control [29]. A characteristic feature of this type of problems is the imposed limitations on the final position of the controlled system. An auxiliary functional, which characterizes the measure of deviation of the system from its final condition, is introduced so that these limitations can be observed:

$$J = pI + q(\bar{\omega} - \bar{\omega}_E)^2, \quad p > 0, \ q > 0. \tag{4}$$

For the solution of the so-formulated problem the class of functions to which the control should belong is stated in advance, $u(\bar{\omega})$. In this study, the governing function belongs to the multitude of the spline functions of the third order with a continuous second product $S_{3,1}(\Delta)$, Δ being the multitude of the units of the spline. It is only natural to assume that an increase in the number of spline units would lead to an increased precision in determining the governing function. Unfortunately, this is not true due to the fact that the formulated problem of optimal control is ill posed. Let us analyze the case in which the multitude of admissible controls is a functional space with steady metrics,

$$\rho(u_k, u_m) = \sup_{\bar{\omega}\in S_{3,1}} |u_k(\bar{\omega}) - u_m(\bar{\omega})|,$$

and $J(u)$ is a continuous functional. In that case, $\forall \varepsilon > 0\ \exists u_1(\bar{\omega}) \in S_{3,1}(\Delta)$ so that (1) $J[u_1(\bar{\omega})] \leqslant J[u_0(\bar{\omega})] + \varepsilon$, and (2) the difference $|u_1(\bar{\omega}) - u_0(\bar{\omega})|$ can acquire random values admissible by the functions $u_1(\bar{\omega})$ and $u_0(\bar{\omega})$ belonging to the multitude of admissible controls. If the function $u_1(\bar{\omega})$ coincides with $u_0(\bar{\omega})$ everywhere except in the interval $(\bar{\omega}_1 + v, \bar{\omega}_1 - v)$, in which the difference $u_1 - u_0$ exceeds a fixed number B, determined by $u_1(\bar{\omega})$ and $u_0(\bar{\omega})$ belonging to the multitude of admissible functions, apparently such $v = v(\delta)$ can be determined for $\delta > 0$ such that the uncoupled equation $|\chi'(\tau) - \chi''(\tau)| < \delta$ is valid for the solutions of the system (3) corresponding to controls $u_1(\bar{\omega})$ and $u_0(\bar{\omega})$. The conditions $J[u_1(\bar{\omega})] \leqslant J[u_0(\bar{\omega})] + \varepsilon$ as well as $\rho(u_k, u_m) > B$ can be satisfied simultaneously by choosing δ to be small enough. The ill-posed problem received can be solved by using Tihonov's method of regularization. According to this method the ill-posed problem is reduced to minimizing the regularizing functional which has the form [30]

$$R^{\alpha}[u]J[u] + \alpha\Omega[u], \tag{5}$$

where $\Omega[u]$ is a stabilizing functional and α is the parameter of regularization. The type of the stabilizing functional depends on the particular problem but most often it is assumed to be

$$\Omega[u] = \int_{t_0}^{t_1} \left\{ c_1(t)u^2 + c_2(t)\left(\frac{\mathrm{d}u}{\mathrm{d}t}\right)^2 + c_3(t)\left(\frac{\mathrm{d}^2u}{\mathrm{d}t^2}\right)^2 \right\} \mathrm{d}t, \quad c_1(t) \geqslant 0, \quad c_2(t) \geqslant 0, \quad c_3(t) \geqslant 0.$$

It depends on the available information on the errors in computing the functional to determine the optimal parameter of regularization. At large values of the parameter α a minimum of the regularizing interval is achieved for functions close to constant.

At small values of α the solution is unstable. When lacking information on the gravity of the errors, a quasioptimal value of the parameter of regularization can be determined by

$$\inf_{\alpha > 0} \left\| \alpha \frac{\mathrm{d}u_\alpha}{\mathrm{d}\alpha} \right\|_{S_{3,1}(\Delta)}.$$

4. NUMERICAL ALGORITHM

Various numerical algorithms analyzed in detail in reference [29] have been elaborated for the solution of the formulated problem of terminal control but they are all based on the polynomial interpolation of the governing function. The method of local variations [18] is used in this study to determine the governing function; it has the following advantages: low sensitivity to the choice of initial approximation; good adaptability to all problems of optimal control; ease of taking into account all kinds of limitations imposed on the phase variables and the governing function; high effectiveness with additive functionals, although there is no problem in applying it to other kinds of functionals. The essence of the method of

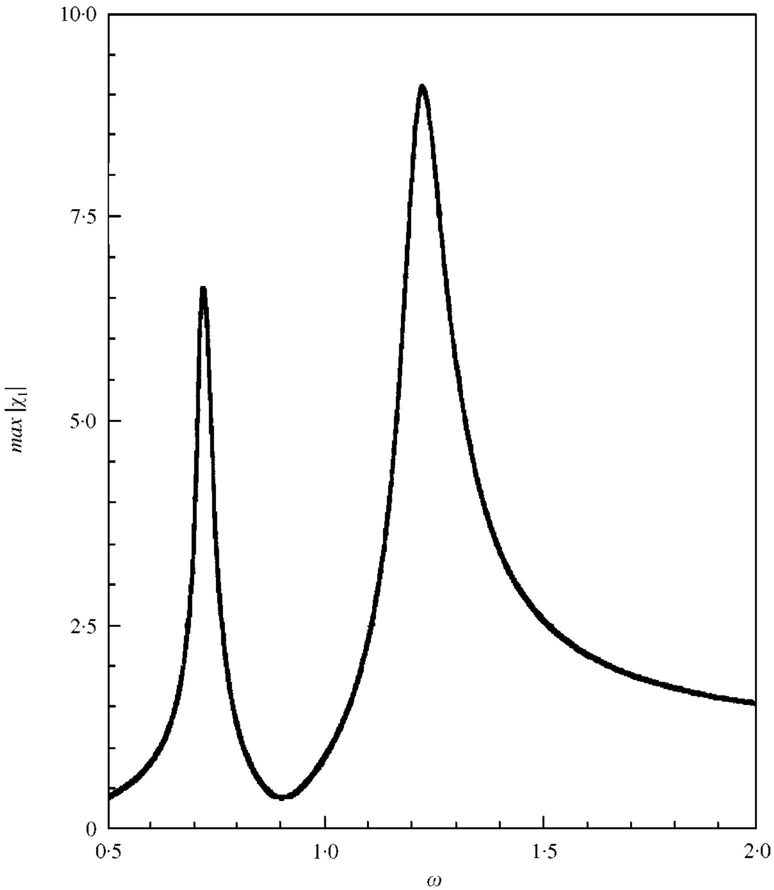

Figure 2. The amplitude–frequency characteristics of the main mass.

local vibrations is the following. The interval within which the governing function has been defined is divided into N subintervals (most often with a constant length)

$$\bar{\omega}_i = \bar{\omega}_0 + \mathrm{i}\Delta\bar{\omega}, \quad \Delta\bar{\omega} = (\bar{\omega}_E - \bar{\omega}_0)/N, \quad i = 0, 1, 2, \ldots, N,$$

and the peak points of each subinterval are used as units of the cubic interpolation spline. Thus, the functional to be minimized is reduced to one functional with $N + 1$ variables, and the problem of optimal control to a problem of non-linear programming. The values of the governing function in the units of the spline are the unknown parameters.

5. RESULTS

The governing function minimizing the regularizing function (5) has been determined for the following parameters of the system: $\mu = 0{\cdot}3$, $\theta = 0{\cdot}95$, $\eta_1 = 0{\cdot}0$, $\eta_2 = 0{\cdot}1$, $\alpha = 0{\cdot}7$, $\beta = 0{\cdot}7$, $\rho = 0{\cdot}2$. In this case, the average angular velocity of the rotor in the antiresonance region is $\bar{\omega}_k = 0{\cdot}9047$. The amplitude frequency characteristics of the main mass are shown in Figure 2. The coefficients p and q in the expression of the functional have the values:

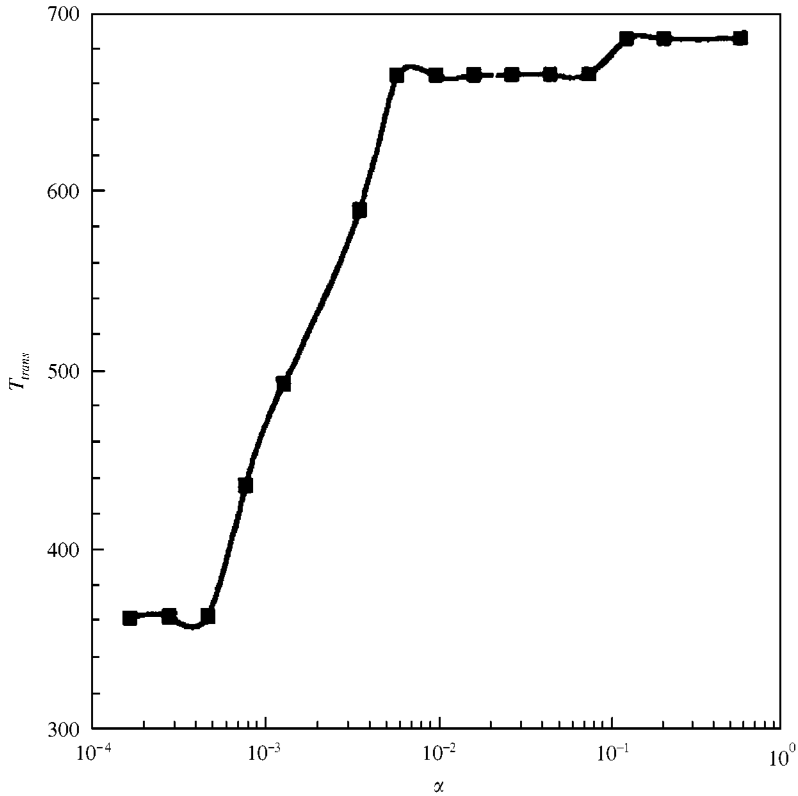

Figure 3. Transition process versus time.

$p = 1{\cdot}0$, $q = 100{\cdot}0$. The system of differential equations is solved with local precision $\bar{\varepsilon} = 1{\cdot}0 \times 10^{-8}$. The governing function is defined for the interval $\bar{\omega} \in [0{\cdot}0, 1{\cdot}0]$. A stabilizing functional of the kind

$$\Omega[u] = \int_{t_0}^{t_1} c_3(t)(\mathrm{d}^2u/\mathrm{d}t^2)^2\,\mathrm{d}t, \quad c_3(t) = 1{\cdot}0$$

is used to determine the regularized solution. The following procedures for determining a quasi-optimal value of the parameter of regularization are used. A sequence of values of the parameter of regularization is assumed in the form of a geometric series $\alpha_{s+1} = q\alpha_s$, $q = \text{const} < 1$, where α_0 is sufficiently large so that the control function $u_{\alpha_0}(\omega)$ is a constant. For each element of this sequence, the corresponding control function is determined after minimizing the regularizing functional $R^{\alpha_{s+1}}[u_\alpha] = I(u_\alpha) + \alpha_{s+1}\Omega(u_\alpha)$, $s = 0, 1, 2$, where, for initial *approximation*, $u_{\alpha_s}(\omega)$ *is assumed*. Here we take $\alpha_0 = 1{\cdot}0$, $u_{\alpha_0} = \text{const} = 0{\cdot}6612145$ and $q = 0{\cdot}6$. In Figure 3 the relation between the time of the transient process, T_{trans}, and the parameter of regularization, α, is shown. For a quasi-optimal value, $\alpha_{opt} = 4{\cdot}70185 \times 10^{-4}$ is taken. The regularized solution is defined for spline units $n = 21$. The results from the minimization of the regularizing functional are illustrated in Figure 4. It can be seen that the time of the transient process depends mostly on the value of the control function in the interval (0·7, 1·0). For $u(\omega) = \text{const} = 0{\cdot}6612145$ (before the beginning of the optimization process), $T_{trans} = 717{\cdot}956$, and after the performed optimization procedure, $T_{trans} = 362{\cdot}121$,

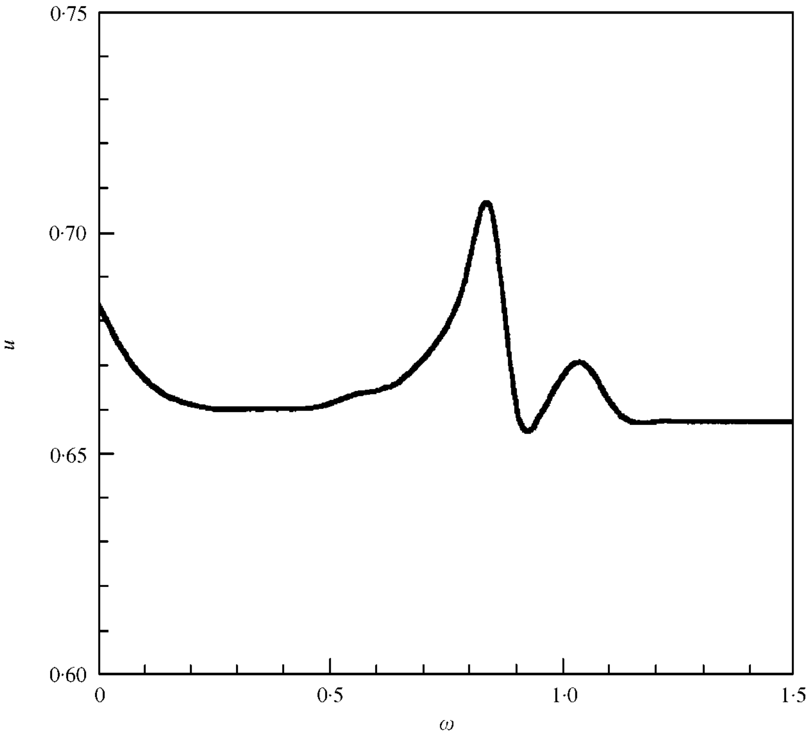

Figure 4. u versus ω.

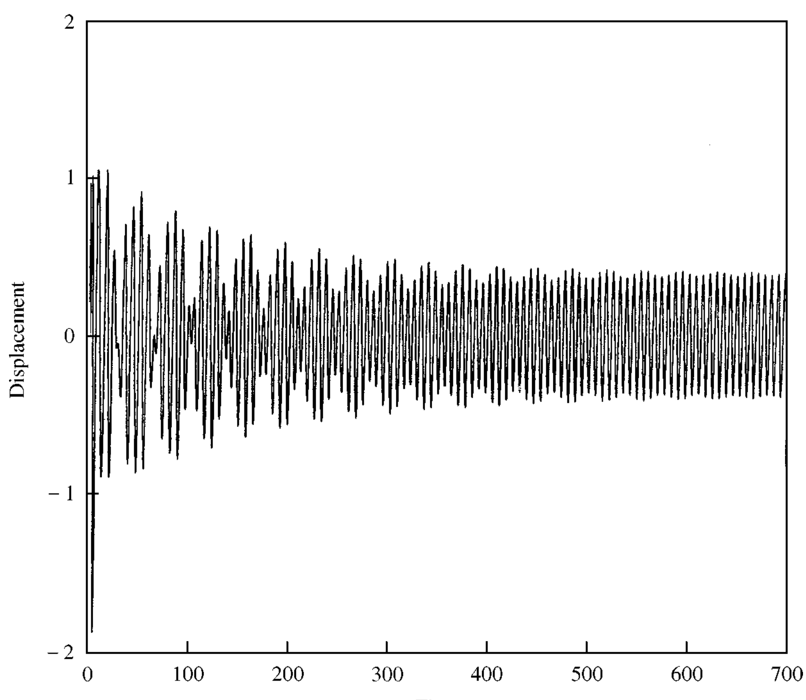

Figure 5. Displacement versus time (without control) taking $u = const = 0{\cdot}6612145$ and $T_{trans} = 719{\cdot}956$.

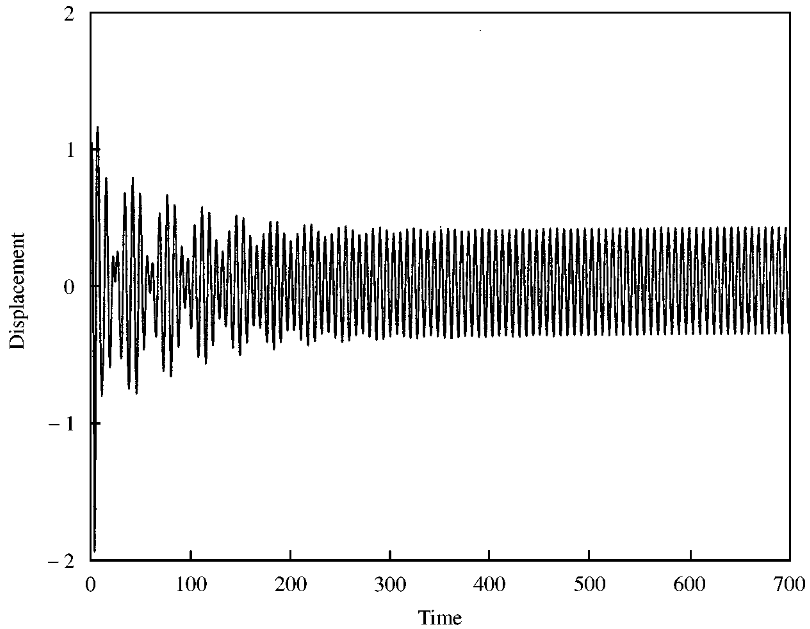

Figure 6. Displacement versus time (with control) $u = u_{opt}$ and $T_{trans} = 362{\cdot}121$.

i.e., the time of the transient process decreases almost two times. The motions of the main mass before the optimization process ($u(\omega) = \mathrm{const} = 0{\cdot}6612145$) and after, for the quasi-optimal value of the parameter of regularization are shown in Figures 5 and 6.

6. CONCLUDING REMARKS

This paper examined a vibrating system with 2d.o.f., subjected to a power disturbance by a non-ideal energy source (direct current motor). An optimal law of control of the motor is synthesized in which the system passes through the first resonant peak into the antiresonance region with minimum amplitude of the main mass. The ill-posed problem is solved by Tikhonov's method of regularization. The obtained law of control of the process of passage of a dynamic system with 2d.o.f., including a limited power supply, depends on the choice of parameters of the system and has to be predetermined should they be altered.

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